# Seismic Demand for SDOF System --in Relating to Seismic Design

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#### Abstract

In traditional structural seismic design procedures, the design earthquake is described by a total design base shear. Considering the inelastic structural response, the elastic base shear is divided by a reduction factor. The displacement ductility is the most commonly used damage criterion to date and is used to estimate the inelastic design response spectra for the traditional earthquake-resistant structural design. However, the displacement ductility can not appropriately describe the low cycle fatigue failures of structures that caused by moderate earthquake ground motions. Proper damage criteria used to define the limit state of structures to the critical seismic excitations should be considered in the nonlinear seismic response analysis. A comprehensive evaluation of design (demand) parameters was performed in this paper for Elastic-Perfectly-Plastic(EPP) Single-Degree-Of-Freedom(SDOF) systems. The Park & Ang's damage model was used as the "Structural Damage Control Model" and was implemented to discuss the seismic design parameters. Because of the implement of the specified damage model the demand parameters include not only the maximum displacement but also the hysteretic energy dissipation. The energy design concepts could be automatically incorporated into the traditional structural seismic design parameters. The seismic records collected from different site conditions in Taiwan area were used as the analysis data base and the present results were compared with the seismic design code in Taiwan. The analyses on the seismic demand parameters that incorporate with the Park & Ang's damage model, were concluded, can verify the building code provisions. The results are more reliable for engineering purpose and can be applied in the traditional seismic design procedures.

#### 1. Introduction

Equivalent lateral force procedures are used in most structural seismic design (Ex. Uniform Building Code). In such procedures, the design earthquake is described by a total design base shear. The base shear is primarily relating to the peak ground acceleration (Z or PGA) of design earthquake, the smoothed elastic design response spectrum (C), and the reduction factor (R). Although all design parameters in this procedure are simple and meaningful in practical structural design, more studies need to be conducted to ascertain the effects and relationships among these design parameters. These works need to predict the response of structure to the extreme ground motions and usually involve the definition of structural limit state under cyclic actions.

The definition of structural damage or collapse is a basic and complex problem in the estimation of seismic design parameters. The displacement ductility, defined by the maximum plastic excursion and the specified structural ultimate value, is the most

commonly used damage criterion to date and is used to estimate the inelastic design response spectra for the traditional earthquake-resistant structural design. However, the displacement ductility by itself can not properly define the degree of structural damage under earthquake ground motions that generally induce the structural low cycle fatigue failure. The energy-based design concepts, first proposed by Housner (1956), should be taken into consideration in the definition of structural limit state under moderate earthquake loading.

Cosenza, Manfredi and Ramasco (1993) examined various structural damage functionals under significant earthquake records. They concluded that the results of Park and Ang's damage functional is similar to the others (Banon and Veneziano and the linear cumulative law of plastic fatigue). The energy design concept, on the other hand, can be automatically incorporated into the traditional structural seismic design procedures through the use of Park and Ang's damage model.

The objective of this paper is to examine the

seismic demands of Single-Degree-Of-Freedom (SDOF) system. These structural demands include the yield strength, allowable displacement ductility, and the reduction factor. A comprehensive evaluation of these design parameters was performed for Elastic-Perfectly-Plastic(EPP) SDOF system. The Park & Ang's damage model was used as the Structural Damage Control Model; then, seismic design parameters were estimated. With the implementation of such damage control model the yield base shear coefficient can be estimated. Discussion on codeprovided reduction factor and the yield base shear coefficient are made. It is observed that the allowable ductility ratio could be used to reflect the effects of low-cycle fatigue failure of the structural system.

# 2. Energy equation

It has been widely recognized by previous researchers that the level of structural damage due to earthquakes does not depend only on displacement ductility but also on the cumulative damage resulting from numerous inelastic cycles. The input energy and/or hysteretic energy is related to the cumulative damage potential of ground motions. Therefore, the energy equation is necessary to be derived to develop reliable design parameters. The equation of motion of a viscous damped SDOF system subjected to a base excitation is given as

$$m\ddot{v}_t + c\dot{v} + f_s = 0 \tag{1}$$

where, m = mass, c = viscous damping coefficient,  $f_s = \text{restoring}$  force,  $v_t = v + v_g = \text{absolute}$  displacement of the mass, and,  $v_g = \text{base}$  (ground) displacement. Integrating equation (1) with respect to v, the energy equation can be obtained as

$$\frac{m\dot{v}_t^2}{2} + \int c\dot{v} \,dv + \int f_s \,dv = \int m\ddot{v}_t \,dv_g \quad (2a)$$

or in short terms

$$E_k + E_d + (E_s + E_h) = E_i$$
 (2b)

where, the RHS term is the seismic input energy  $(E_i)$ , the three terms of the LHS are the kinetic energy  $(E_k)$ , the damping energy  $(E_d)$ , and recoverable elastic strain energy  $(E_s)$  and irrecoverable hysteretic energy  $(E_h)$ , respectively. Equation (2) is the so-called absolute energy equation (Uang and Bertero, 1990).

Considering the energy-based design method, a satisfactory design of structure implies the energy supply  $(E_d + E_h)$  should be larger than the energy demand  $(E_i)$ . Just as the displacement ductility, the energy demand spectra alone are not sufficient for

conducting reliable design of structures. Furthermore, how to implement the energy spectra into the practical design procedures is still a problem. In other words, to develop practical design methods that can take the displacement ductility as well as the energy demands into considerations is necessary.

# 3. Damage parameters and damage criteria

Structures are designed to against the seismic actions, no collapse is allowable under moderate earthquake. In design practices, there are many structural response quantities can be considered as damage parameters. These parameters can, then, be used to construct proper damage criteria for structures. The displacement ductility  $\mu_d$  and hysteresis ductility  $\mu_e$  were frequently used and defined as

$$\mu_d = \frac{v_{max}}{v_v} \tag{3a}$$

$$\mu_e = \frac{E_h}{f_v \cdot v_v} + I \tag{3b}$$

where,  $f_y$  = yielding strength,  $v_y$  = yielding displacement, and,  $v_{max}$  = maximum displacement under ground motion excitation.

One of the widely used damage model is the Park & Ang's model (1985) which was defined as

$$D_{PA} = \frac{v_{max}}{v_{utl}} + \beta \frac{E_h}{v_{utl} f_y}$$
 (4a)

or

$$D_{PA} = \frac{\mu_d + \beta (\mu_e - 1)}{\mu_{d,uil}}$$
 (4b)

where,  $\mu_{d,utl}$  = ultimate displacement ductility due to monotonic loading,  $\nu_{utl}$  = ultimate displacement due to monotonic loading,  $\beta$  = a structure relating parameter. The  $\beta$ -value, which is dependent on the post-yielding behavior of structures, is completely independent of the loading history. The better the behavior of post-yielding of structures the smaller the values of  $\beta$ . The Park and Ang's model is a linear combination of displacement ductility and hysteresis ductility, and is not a normalized damage functional.

For a specified time history of ground motion, the cumulative damage can be calculated. A sample of time histories of  $D_{PA}$  with different  $\beta$ -values are shown in Fig. 1. Those of another type of cumulative damage law  $D_F$  (Jean, 1996), which takes into account the different amounts of plastic displacement are also shown in Fig. 1(b) for comparison. The results are

quite similar for some parameter pair, say,  $\beta = 0.2$  and b = 1.6. Both damage functionals are similar to hysteretic energy  $(E_h)$  (shown in Fig. 1(a)) in shape and are believed to be applicable for engineering practices. The Park and Ang's damage model was used to estimate the seismic design parameters (demands) in this paper.

It should be noted that (1) the value of  $\mu_{d,ut}$  is 4 ( $v_{utt} = 4 v_y$ ) in this example, so, it means no damage for the case of  $D_{PA} < (1 / \mu_{d,ut}) = 0.25$ ; (2) for the case of  $D_{PA}(\beta=0.0)$ , which was used in the traditional seismic design, there is no damage in this example because  $D_{PA}$  is less then 1.0; (3) for the cases of  $D_{PA}(\beta \ge 0.1)$  and  $D_F(b=1.6\sim 1.8)$ , however, the structural damage was occurred due to the effects of low cycle fatigue.

# 4. Seismic demands based on damage control

In practical procedure, the ultimate displacement ductility  $\mu_{d,utb}$  the parameter  $\beta$  and the level of damage  $(D_{PA})$  are assumed to be prescribed. For each ground motion record, given a  $f_y$  value, the maximum

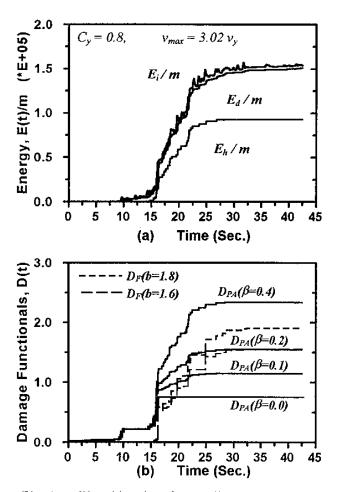


Fig. 1 (a) Time histories of normalized cumulative egergy, and (b) damage functionals  $\mu_{d,utl} = 4.0$ 

(allowable) displacement  $v_{max}$  (or  $v_a$ ) and other response quantities can be calculated from eq. 2 and 4. The seismic demand quantities can, then, be obtained through the use of damage functional. A schematic flow chart of this procedure is shown in Fig 2.

The process of determining the yield level for specified ductility ratio (or damage functional value) should be done very carefully because the relationship between yield level and ductility demand  $(f_y - \mu_d)$  is not necessarily a monotonic function. For most earthquake records, there exist many yield levels that result in a specified ductility demand. Clearly, only the highest of these yield levels that defines the strength demand, the other(s) should be ignored. (Nassar and Krawinkler, 1991) Then, a monotonic relationship between damage functional and yield strength demand can be expressed, for example, as

$$D_{PA} = D_{PA} (f_{v}, \mu_{d.uil}, T, \beta, ...)$$
 (5)

From eq. (5), relationship between any two of the parameters (the others were prescribed), shown in eq. (5), can be constructed. As an example, for specified values of  $\mu_{d,urb}$  T and  $\beta$ , ..., the strength demand can be obtained by inverting eq. (5) and is expressed as

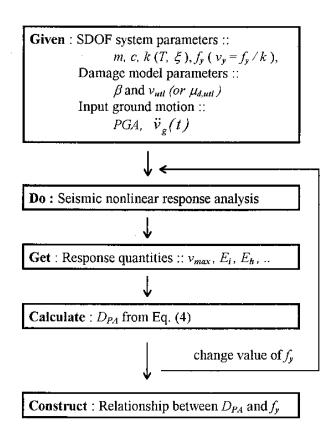


Fig. 2 A schematic flow chart for damage calculation,  $D_{PA} = D_{PA} (f_{y, ...})$ 

$$f_{v} = f_{v} \left( D_{PA}, \mu_{d,ud}, T, \beta, \ldots \right)$$
 (6)

other demands can be obtained by the same procedure.

As mentioned before, the ultimate ductility capacity,  $\mu_{d,utl}$ , of structure is assumed, the design objective is to provide sufficient strength capacity so that the ductility demands do not exceed the capacities of structure by an adequate margin of safety during the design earthquake. Furthermore, the damage level of structures is controlled by the value of  $D_{PA}$ . The dimensionless seismic demands presented in this paper are for prescribed target ductility,  $\mu_{d,utl}$ . These parameters are defined as the followings

$$C_{y} = \frac{f_{y}(D_{PA}, \dots)}{PGA \cdot m}$$
 (7a)

$$R = \frac{f_e}{f_y(D_{PA}, \dots)}$$
 (7b)

where,  $f_y$  is calculated from eq. (6) for specified values of  $D_{PA}$  and other parameters;  $C_y$  and R are yield base-shear-coefficient (BSC) and reduction factor, respectively;  $f_e$  is maximum BSC of elastic response to the same ground motion (PGA is normalized to 1g). In the traditional seismic design procedures, the yield BSC was obtained by  $C_y = f_e / R$ .

The spectra of dimensionless design parameters,  $C_y$  and R, can be generated from eqs (7) through statistical analysis. It should be emphasized that the energy design concept was automatically introduced into eqs. (7) with the use of Park and Ang's damage model. Therefore, the energy demand  $(E_h)$  were implicitly included in these spectra; the spectra calculated by the procedures mentioned above are more reliable and can be used in the traditional seismic design procedures, straightforwardly. It is not necessary to change the procedures of the traditional seismic design.

# 5. Data base, structural model and design code for Taipei basin

Four records collected in Taipei basin, Taiwan,

were used to study the effects of damage model on seismic demands. The basic information of these records is shown in Table 1. It is quite clear that the site condition of these stations is very soft from engineering point of view.

The structural model was simplified as a Elastic-Perfectly-Plastic Single-Degree-Of-Freedom (EPP-SDOF) system with 5% structural damping ratio. The Newmark - $\beta$  constant acceleration method ( $\gamma = 1/2$ ,  $\beta = 1/4$ ) was used for nonlinear dynamic analysis.

The seismic design code for Taiwan was modified recently. The basic form was expressed as

$$V = \frac{ZI}{\Omega \alpha_v} \left(\frac{C}{F_u}\right)_m W \tag{8a}$$

$$= \frac{ZI}{\Omega \alpha_{y}} C_{y} W$$
 (8b)

where, Z = 475-years-return-period design PGA; I = important factor;  $\Omega$  and  $\alpha_y =$  structure related coefficients; C = elastic design spectrum (base shear coefficient);  $C_y =$  yield-base-shear-coefficient spectrum;  $F_u =$  reduction factor. The reduction factor is of the Newmark's formats, and, therefore, is a function of structural period and structural ductility (an allowable ductility is introduced for calculation of reduction factor).

#### 6. Seismic design parameters

Based on the procedure and data base mentioned above, average spectra of seismic demands can be obtained. Some of the results are represented and discussed in the following sections.

#### 6.1 Strength reduction factor

The strength reduction factor, as defined by eq. (7b), is the ratio of spectral ordinates of the elastic and inelastic strength demand spectrum. This factor shows how much the yield BSC demand of a given elastic SDOF system can be reduced, by allowing the system to behave inelastically, within the limit of a predefined

Table 1. Primarily data base (recorded in Taipei basin) used in this study

Station Code	Recording Date	Peak Ground Acceleration	Predominate Frequency (Hz)
CKS, 4010L	Nov. 15, 1986	96.304 (gal.)	1.392 Hz $(T_P = 0.719 \text{ Sec.})$
CKS, 4010T	Nov. 15, 1986	79.895 (gal.)	$0.928  Hz  (T_P = 1.078  Sec.)$
TAP037E	Jun. 25, 1995	61.126 (gal.)	$1.074 \; Hz  (T_P = 0.931 \; Sec.)$
TAP037N	Jun. 25, 1995	75.918 (gal.)	$1.074 \; Hz  (T_P = 0.931 \; \text{Sec.})$

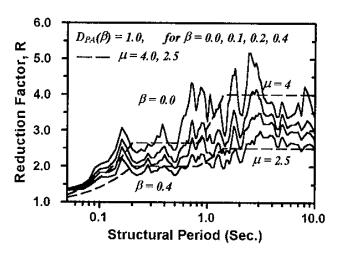


Fig. 3 Spectral plot of reduction factor

damage level (also the ultimate ductility ratio).

Considering the structural damage at a level of limit state of collapse ( $D_{PA} = 1.0$ ), the strength reduction factor was shown in Fig. 3 for cases of  $\beta = 0.0$ , 0.1, 0.2, and 0.4, respectively. It somewhat follows the Newmark's format. The spectra of reduction factor with Newmark's format were also shown in the same figure for comparing (dashed lines, for  $\mu = 4$  and 2.5, respectively). Figure 3 shows that the strength reduction factor depends strongly on the period and damage model ( $\beta$ -value), also on the ductility demand (Jean, 1996).

It is observed that (1) the increase of  $\beta$ -value in  $D_{PA}$  will reduce the value of reduction factor; (2) a reduced ductility (allowable ductility) should be used to estimated the ductility-dependent reduction factor for structures failed by low-cycle fatigue ( $\beta > 0.0$ ). This observation of reduction factor will be discussed again later.

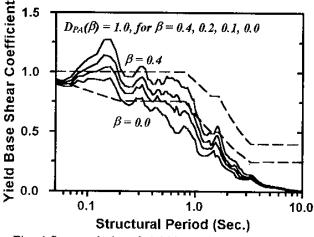


Fig. 4 Sprectral plot of yield base shear coefficient (dot line: obtain from Taiwan design code with  $R_a = 1.5$  and 2.0, respectively)

# 6.2 Strength demand

Strength demand is expressed here as a base shear coefficient (eq. (7a)). The inelastic strength demand spectra represent the period dependent yield level of a structural system. As a matter of fact from this study the Yield BSC can be derived under damage control model ( $D_{PA} = 1.0$  with  $\beta = 0.4 \sim 0.1$ ), as shown in Fig. 4 It should be noted that the values of BSC of the case with  $\beta = 0.2$  is about 20% higher, for some period range, than those of the case with  $\beta = 0.0$ . The later case is used in traditional seismic design (no hysteretic energy ( $E_h$ ) is considered). Neglect the low-cycle fatigue damage on the analysis of structural damage is inadequate.

The effects of ultimate ductility on the yield BSC was shown in Fig. 5 for cases of  $D_{PA} = 1.0$  with  $\beta = 0.2$  and  $\mu_{d,utl} = 1$ , 2, 3, 4, 5 and 6, respectively. The case of  $\mu_{d,utl} = 1$  (the upper line in Fig. 5) is the elastic spectrum, which shows many peaks. In the inelastic spectra the peaks diminish and essentially disappear for larger ductility ratio; the shape of the elastic and inelastic strength demand spectra are not necessarily similar, and are in fact rather dissimilar if the elastic spectrum has steep peaks.

Comparing to the reduction factor spectra shown in Fig. 3, it can be seen that the peaks and valleys in the elastic strength demand spectrum usually coincide with those of the strength reduction factor spectrum. This the reason why the inelastic strength demand spectrum is much smoother than the elastic one. Thus, the yield BSC developed from the elastic strength spectrum divided by the strength reduction factor, as adopted in the many seismic design codes, may be inadequate to the seismic design for site condition similar to that of Taipei basin. A more reliable design

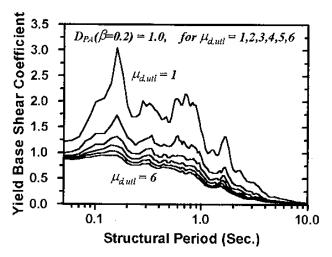


Fig. 5 Sprectral plot of yield base shear coefficient for ultimate ductility from 1 through 6

yield BSC should be estimated directly from the inelastic strength demand spectrum as shown in Fig. 5 for different ductility ratio.

# 6.3 Important factor

The important factor is used in many seismic design codes to increase the design BSC level for essential and hazardous facilities. It was recognized that higher force levels alone do not necessarily improve seismic performance. For structures subjected to seismic excitations, it was experienced that the structures would be repairable for cases of  $D_{PA}$  less than 0.5 (De Leon and Ang, 1994).

It is assumed in this paper that the damage of essential facilities will be controlled to the level of  $D_{PA}$  = 0.5 due to the design earthquake, and  $D_{PA}$  = 0.75 for hazardous facilities. The strength demand needed to control the structural damage to a level mentioned

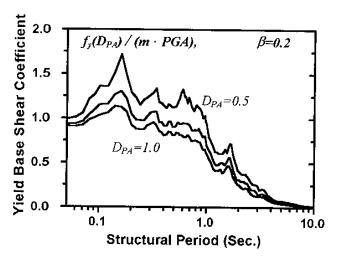


Fig. 6 Comparison of yield base shear coefficient for different damage levels (0.5, 0.75, 1.0)

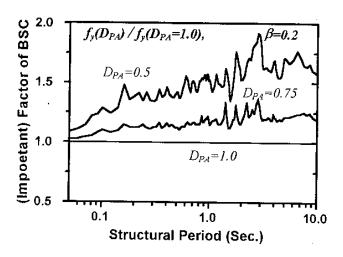


Fig. 7 Factors of base shear coefficient needed for different damage levels (0.5, 0.75, 1.0)

above were shown in Fig. 6 (for  $\beta = 0.2$  and  $\mu_{d,utt} = 4$ ). It was noted that the strength demand for case of  $D_{PA}$  = 0.5 is quite similar to that for case of  $D_{PA}(\mu_{d,utl} = 2)$  = 1.0 as shown in Fig. 5. It is similar, too, for case of  $D_{PA} = 0.75$  and  $D_{PA}(\mu_{d,utl} = 3) = 1.0$ .

The factors of strength demand needed to control the damage level to  $D_{PA} = 0.5$  and  $D_{PA} = 0.75$  were shown in Fig. 7 (estimated directly from Fig. 6). These factors can be compared with the important factors used in the design code. The results show that the (important) factor are period-dependent. The strength demand needed to control the structural damage to a prescribed level can be found from Figs. 6 and 7. These figures provided another way to study and estimate the important factor for seismic design.

### 6.4 Allowable ductility

The allowable ductility,  $\mu_a$ , is defined as the maximum ductility that can be allowed to against the damage of structure due to design earthquakes. Therefore, the form of  $\mu_a$  is same as that defined in eq. (3a). For specified damage model and damage level, the allowable ductility can be obtained. Figure 8 shows the relationship between allowable ductility and ultimate ductility for damage model with  $\beta = 0.2$  and structural period  $T_S = 0.2$ , 0.5, 1.0, and 1.5 Sec., respectively. The relationship between  $\mu_a$  and  $\mu_{d,utl}$  is almost linear as shown in Fig. 8(for  $\mu_{d,utl} \le 6.0$ ). (The relationship becomes not stable for cases with  $\mu_{d,utl} > 8.0$ ).

Define a reduced-ductility factor,  $\Delta$ , as

$$\Delta = \frac{D_{PA} \cdot \mu_{d,utl} - I}{\mu_a - I} \tag{9}$$

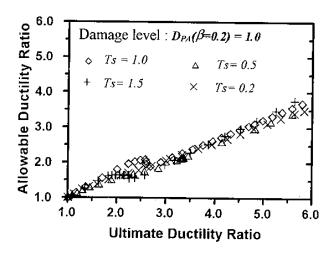


Fig. 8 Relationship between allowable ductility and ultimate ductility

obviously,  $\Delta$  can be a function of  $\beta$ , if  $D_{PA} = 1.0$  was assumed for the damage control model.

The reduced-ductility factor,  $\Delta$ , is a key-role in seismic design. As mentioned before, the energy design concept can take into account the effects of low-cycle fatigue failure of structures due to seismic loading. The implementation of energy design concept can be done through the used of factor  $\Delta$  and, then, the energy-based design parameters can be obtained and applied to traditional seismic design.

The value of the damage-model-dependent factor  $\Delta$  was shown in Fig. 9 for Taipei site. That of other site conditions were shown in the same figure for comparison. It shows that (1) the relationships between  $\Delta$  and  $\beta$  is quite stable; (2) the value of  $\Delta$  is higher for Taipei basin due to the local site condition; however, (3) there were no significant difference for other site conditions (soft and hard site). For a typical structure system, the  $\beta$ -value in damage model  $D_{PA}$ 

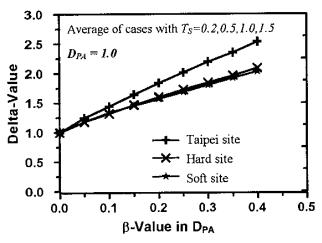


Fig. 9 Relationship between reduction of ductility,  $\Delta$  and  $\beta$ -value of Park and Ang's damage model

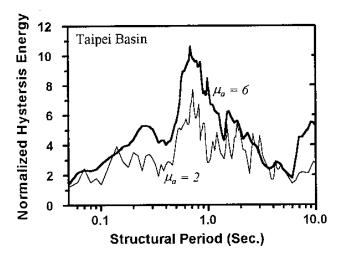


Fig. 10 Spectral plot of index of hystersis energy,  $\gamma$ 

could be obtained by experience. Thus, the  $\Delta$ -value can be estimated from Fig. 9 for prescribed damage level,  $D_{PA}$ . The allowable ductility can, then, be obtained by inverting eq. (9)

$$\mu_a = 1.0 + \frac{\mu_{d,utt} - 1.0}{\Delta}$$
 (10)

It is necessary to use the allowable ductility  $\mu_a$ -value instate of  $\mu_{d,ut}$  in the calculation of strength reduction factor in traditional seismic design procedures.

According to the Taiwan seismic design code eq. (10) was used to calculate the allowable ductility. Then, this allowable ductility was applied in the calculation of strength reduction factor. The  $\Delta$ -value was set as 2 for building design. In other words, the  $\beta$ -value was assumed to be about 0.2 as shown in Fig. 9 for Taipei basin, and much higher for other site. It seems that the  $\Delta$ -value needed to be studied more carefully.

# 6.5 Energy demand

In order to explain how the energy demand was introduced into the design parameters, which were estimated from the proposed procedure in this paper, eq. (4) was substituted into eq. (9) to obtain the following equation

$$\Delta = I.0 + \beta \cdot \lambda \tag{11}$$

where

$$\lambda = \frac{E_h}{f_v(v_{max} - v_v)} \tag{12}$$

is a normalized-dimensionless index of hysteretic energy  $(E_h$ , the energy demand).

This definition of  $\lambda$  is a little different from that of  $\gamma$ , which defined by Fajfar and Vidic (1994). The value of  $\lambda$  controls the relationship between allowable ductility and the ultimate ductility as shown in eqs. 11 and 12. It can be estimated from hysteretic energy for every earthquake record. It should be noted that (1) the  $\lambda$ -value can be directly estimated from Fig. 9 (the slope of  $\Delta$ - $\beta$  relationship curve); (2) the  $\lambda$ -value is damage model ( $\beta$ -value) independent by definition.

The average  $\lambda$ -spectra of Taipei basin were shown in Fig. 10 (for cases of  $\mu_a = 6$  and 2). Again, the  $\lambda$ -spectra (also the  $E_h$ -spectra) are highly period-dependent. It shows a steep peak around structural period  $T_S = 0.7$  Sec.. This is due to the local site condition. The  $\lambda$ -value estimated from Fig. 9 was 3.83 for  $\mu_a = 4$ . This value is reasonable for most period

range, however, under estimated for period near 0.7 Sec.. Consequently, the allowable ductility will be over estimated (eqs. 10 and 11) and the strength demand will be under estimated.

#### 7. Conclusions and discussions

Based on this study, there were some concluding remarks could be drawn:

- The damage functionals should be used to defined the damage level of structures. Then, the effects of low-cycle-fatigue due to design earthquake or the energy design concepts can be incorporated into the design parameters.
- (2) An allowable ductility should be introduced for the estimation of reduction factor. The allowable ductility can be obtained from the normalized-dimensionless hysteretic energy spectra.
- (3) Normalized hysteretic energy is a good index to modify the ductility capacity of structures. The proposed dimensionless parameter, γ, can be used to find the allowable ductility that can, then, be used for the calculation of strength reduction factor.

It must be emphasized that this study focuses only on a small part of a big problem. The seismic demands are evaluated only for selected ground motions in Taipei basin. Much more works needs to be done in the context of demand evaluation for seismic design, especially, the definition of structural damage. If a proper damage model could be found, the more reliable damage-control-based design parameters could, then, be obtained by the procedures proposed in this paper.

## 8. Acknowledgment

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